THE APPORTIONMENT OF REPRESENTATIVES

ANNUAL ADDRESS OF THE PRESIDENT

WALTER F. WILLCOX
Cornell University

According to an unbroken series of precedents running back to the origin of the American Economic Association, some thirty years ago, the President selects for the theme of his annual address either a fundamental economic problem or some phase of the relation between economics and our social, political, educational, or legal institutions. In breaking with this tradition I am taking a step which requires an opening word of explanation. The reason underlying the precedents and giving them their value is the principle that the man whom the Association has honored should offer in return whatever he has that seems most worthy of attention from the Association and the public. But of that he must be the judge. In the present case exceptional circumstances seem to justify an unprecedented choice. A brief statement of the circumstances may serve to secure for the subsequent argument a more indulgent attention.

The presidents of a score of American universities were invited some years ago by the Census Office to nominate graduate students of economics, one from each institution, who might assist in the study of various problems within its field of work. The Secretary of the Economic Association, who was temporarily in the employ of the Federal Government, had suggested that the invitation be given and was responsible for the results. His position had grown naturally out of his secretarial duties and was intended as a recognition of an extended report and accompanying papers just published by a committee of the Association.[1]

Somewhat more than a year after this group assembled in Washington the results of the census of 1900 laid the basis for the apportionment law of 1901 and members of the group prepared for Congress the tables around which the prolonged discussion centered. While following these congressional debates they became interested in certain vexatious difficulties and anomalies in the method of apportionment, for the solution of which

[1] *The Federal Census, Critical Essays by Members of the American Economic Association collected and edited by a Special Committee,* 1899.

Congress naturally could not wait. So it cut the knot and turned to other business. The decision which Congress reached was seemingly devoid of justification in theory and had been rejected by all trained students who had faced the problem. Twenty years earlier, our first president, General Walker, had characterized a similar solution as losing "all hold on any principle governing the matter."

During the following decade members of the group continued a spasmodic discussion of the apportionment problem among themselves and with others, and, in 1910, when it again became urgent, a solution had been worked out which could hardly have occurred to any one of the group alone or to any member of Congress. The solution commended itself to the House Committee on the Census, aroused no opposition in House or Senate, and was speedily followed in the law. The fact that these members of our Association have contributed to elucidate the difficult problem may justify the choice of a theme remote from the field of economics as traditionally interpreted.

A few months before the group was brought together the president of the Association, in closing his annual address, had said: "I believe that the largest opportunity of our economists in the immediate future lies not in theories but in practice, not with students but with statesmen, not in the education of individual citizens, however widespread and salutary, but in the leadership of an organized body politic.[2]

It is somewhat in the spirit of President Hadley's speech at New Haven that I ask your attention to this theme. My immediate object is to present the reasons for believing that the problem of apportionment under the provisions of the Constitution, a technical problem to which at intervals through more than a century the attention of Congress, executive officers, and statistical experts had been directed, with only temporary and provisional success, has at length been solved by reconciling the demands of Congress with the conditions imposed by theory. My remote object is to show that the ground has been cleared for renewing under more favorable auspices that effort to check the steady and rapid increase in the size of the House of Representatives which was made in 1850 and abandoned in 1870.

Only two provisions of the Constitution bear on the present problem, one that "representatives . . . shall be apportioned among the several states . . . according to their respective num-

[2] A. T. Hadley, Am. Econ. Assn., *Economic Studies,* IV (1899), p. 28.

bers," and the other that "each state shall have at least one representative."

Between 1790 and 1911, inclusive, two fundamentally different methods of apportionment were employed, one method in seven apportionments and the other in six. It is theoretically possible, of course, that the two methods may be equally good or equally bad, but probably one method is better than the other and possibly one method is wrong and the other right. The two methods differ in that one, which I will venture to call the method of an assumed ratio, starts from an assumed number of persons to one representative, the number being usually but not necessarily a whole number of hundreds or thousands; and the other, which, but for the fact that it has long been called the Vinton method from the name of the representative who introduced it, might be named, in contrast to the first, the method of a computed ratio, starts from an assumed number of representatives in the House and computes the average number of persons to a representative by dividing the constitutional or representative population of the United States by that number. The quotient is then used as a divisor for the population of each of the states, a representative is apportioned for each unit in the resulting series of quotients, the sum of which under all ordinary circumstances is less than the number of representatives to be apportioned, the remainders are arranged in order of size, and a representative assigned to each in succession until the desired total is secured. This method sometimes leaves one or more major fractions, or fractions larger than one half, without an additional representative and sometimes gives an additional representative for one or more minor fractions, results which have been serious stumbling-blocks to Congress but are inherent in the method.

The Vinton method was used at every census between 1850 and 1900, inclusive; the method of an assumed ratio was used at every census between 1790 and 1840, inclusive, and again in 1910. The latter method has taken two forms, one in which all fractions after division are disregarded in the apportionment, and the other in which all fractions larger than one half entitle the states wherein they occur to an additional representative. These two may be called the method of rejected fractions and the method of major fractions.

After this preliminary clearing of the ground my main thesis may be stated as follows: The method of major fractions is the correct and constitutional method of apportionment.

To determine what is the correct and constitutional method a test of correctness is needed. The requirement that representatives shall be apportioned among the several states according to their respective numbers cannot be strictly and exactly met. Probably no two states ever have had or ever will have exactly the same number of inhabitants to one representative. This fact does not authorize Congress to disregard the constitutional mandate, but does oblige it to interpret the phrase by reading into the Constitution at this point the words "as near as may be," so that the requirement means, representatives shall be apportioned among the several states according to their respective numbers as near as may be. This is the substance of the claim made by Daniel Webster in his committee report to the Senate in 1832,[3] which convinced that body and ten years later convinced both Houses that the method of apportionment which had been followed by Congress theretofore was unsound, if not unconstitutional. A bill drawn in conformity with Webster's position reached President Tyler in 1842. He finally signed it, but at the same time filed a memorandum expressing doubts regarding the constitutionality of the innovation. Since Tyler's day no serious dissent from Webster's position has been expressed, I believe, either on the floor of Congress or elsewhere, and it may be accepted as unchallenged. It affords one criterion whereby the apportionment of representatives to any state may be tested. If there is doubt, for example, whether State A is entitled to ten or eleven representatives, three ratios are needed: (1) the number of persons to one representative in State A with ten representatives, (2) the same in State A with eleven representatives, (3) the same in the United States. The nearness of the first and second ratios to the third, or standard, is then measured, whichever ratio is nearer to the standard is accepted, and State A assigned the corresponding number of representatives.

This is the test and the only test which can be derived from Webster's argument. It is of great help, but it does not completely solve the problem.

A second test, to be set beside Webster's, is suggested by the original object of apportionment. This was to give the more populous states the larger representation in the House to which their numbers were thought to entitle them and which they did not receive in the Senate. From the fundamental purpose of ap-

[3] *The Works of Daniel Webster* (Little and Brown, Boston, 1851), III, pp. 369ff.

portionment it follows that a method giving the populous states systematically either more members or fewer members per unit of population than the small states is incorrect.

We have, then, two criteria, of a just and constitutional apportionment: first, the ratio in each state must be as near as may be to the standard ratio; and, secondly, the method must hold the scales even between the large and the small states.

The ratio either for the country or for a state is usually expressed as the average number of persons to one representative. For example, under the apportionment law of 1911 there were 211,877 persons to a representative. But for my purposes it is better to conceive that each inhabitant of the country or of a state has an equal interest in a representative and to express that interest in the form of a decimal, such as .000005 or .0000047 or .00000472. A further improvement gets rid of the long decimals by taking a round number of people, say 10,000,000, as a unit and computing the number of representatives to that unit. When ratios are stated in this form, the meaning is more quickly grasped through ear or eye and comparisons between two ratios are more easily made.

Armed with these criteria of a correct apportionment, we are in a position to approach the problem. Because of the historical interest attaching to it I will first apply the tests to the apportionment of 1832, the last made under the method of rejected fractions. The ratio used in the bill before the House was one representative to 47,000 people, which is equal to 213 representatives to 10,000,000. Under this ratio Massachusetts was entitled to 12.99 representatives and the question before Congress was, Should it receive 12 or 13? If it received only 12 representatives, it would be at the rate of 197 to 10,000,000 people; if 13, it would be at the rate of 213 to 10,000,000 people, the standard ratio. Tried by Webster's test of nearness, then, Massachusetts was entitled to 13 representatives and was defrauded by receiving only 12. It is not surprising that Edward Everett in the House and Daniel Webster in the Senate joined in a vigorous, though in its momentary effect an unsuccessful, onslaught upon a method which led to such results.

They did not notice, however, and since that controversy ended, no one, I believe, has remarked, that the system of rejecting all fractions, even those as large as .99, confers an unjust advantage in the long run upon the populous states, among which at the time

Massachusetts held a leading position. If there are eleven states, one of 10,000,000 and ten of 1,000,000, and all remainders are rejected, the chances are that the ten remainders of the small states will in combination be several times as great as the one remainder in the large state. Or in the case of Massachusetts in 1832, with a ratio of one representative to 47,000 people and 12 representatives, the apportionment against which Everett and Webster protested, that state would have had, to be sure, only 197 representatives to 10,000,000 people, far short of the standard 213; but the smallest five states then in the Union, which together had a population somewhat less than that of Massachusetts, would have received only 172 representatives per 10,000,000 people. Massachusetts would have fallen 16 below the standard, but they would have fallen 25 below Massachusetts. Massachusetts would have had a rejected fraction of .99 but they would have had rejected fractions amounting in combination to 2.38.

To test the conclusion that the method of rejected fractions favors the populous states I have reapportioned the House for every decade between 1790 and 1840 by applying the method of apportionment employed in 1910 to the conditions and compared the results with those actually embodied in the laws. The states which were given too many representatives between 1790 and 1840 include New York and Massachusetts, each of which received four more than it deserved, Pennsylvania, which received three more, and Virginia, which received two more. Among the smaller states Delaware should have had four more than it had, Vermont three more, New Jersey two more, Connecticut, North Carolina, Tennessee and Missouri one more. Thus, during the first five decades of apportionment, the populous states as a group got thirteen representatives more than they were entitled to and the small states, as a group, thirteen less. The efforts of Everett and Webster, called forth by what looked like a serious injustice to Massachusetts, led to the introduction ten years later of a better method, but in so doing overturned a method by which in the long run Massachusetts had profited as much as any state in the Union.

I conclude that the method of apportionment used between 1790 and 1830, under which all fractional remainders were rejected, is incorrect and unconstitutional and that for two reasons: first, it does not apportion in each state as near as may be to the standard and, secondly, it regularly results in over-representing the populous states.

If this conclusion is correct, but two methods of those which have been tried remain to be examined, the method of major fractions and the Vinton method. The arguments in favor of the former and against the latter are of two classes: practical arguments likely to gain the attention of members and committees of Congress; and theoretical arguments appealing to statists and mathematicians, but with little meaning for the general public. During the prolonged controversy the solution reached by Congress has at times appeared to theorists indefensible and the solutions proposed by theorists have seemed to Congress unjust or incomprehensible. In this controversy I believe that Congress has usually been right in its instinctive judgment, but unable or indisposed to furnish a convincing defense of its decision, and that the theorists, while better equipped with arguments, have been defending an untenable position.

The fundamental difference between the Vinton method and the method of major fractions, as already explained, is that the former starts from an assumed number of representatives and the latter from an assumed ratio. So we are brought face to face with the question, Which is the correct point of departure? Certain practical considerations likely to affect the choice of Congress between these methods will first be presented and later the more theoretical arguments.

The method of an assumed ratio is implied in the Constitution, which says, "The number of representatives shall not exceed one for every thirty thousand." The purpose and result of this clause were to limit the size of the House. If the Convention had had in mind a method starting with a number of representatives, it would have been more natural to make the clause read, "The number of representatives shall not exceed x (e.g., 120 or 112)." The phrasing of this limitation indicates that in thinking of apportionment the members of the Constitutional Convention of 1787 instinctively began with a round number of persons to each representative. Yet the form of the limitation was doubtless influenced also by a desire to open the way for an increase in the size of the House as the population of the country grew.

Until long after the fathers of the Constitution had passed off the stage, this method was followed without hesitation or challenge.

When the change to the Vinton method, or method of a computed ratio, was made, it was not from dissatisfaction with the method of an assumed ratio in the form it took after 1840. The main

motives for the change were a desire to end the rapid increase in the size of the House of Representatives and a desire to withdraw a vexatious and contentious question from the arena of public discussion. The way was not then open, as it is now, for accomplishing either purpose if the method of an assumed ratio was retained.

There are two serious difficulties with the results of the Vinton method. It often results in a major fraction which does not entitle the state to another representative; it sometimes results in two consecutive tables in which the total size of the House differs by one, but in the larger House the number of members from some state is one less. The latter possibility first came prominently before Congress in the tables prepared in 1881, which showed that if the House had 299 members Alabama was entitled to 8, but if it had 300 members Alabama was entitled to only 7. Hence this anomaly has come to be known at the "Alabama paradox." When Congress has faced the rejection of one or more major fractions under the Vinton method, it has assigned new members for those fractions in defiance of the method's requirements. When it has faced the Alabama paradox, it has avoided the difficulty by choosing another table from the series. Thus neither of these anomalies has occurred in any actual apportionment.

Under the system of an assumed ratio the number of representatives in a state depends upon the initial ratio and the population of the state, and not at all upon the population of any other state or of the country. Thus, if serious errors had crept into the enumeration of any state and been discovered after an apportionment law had been passed, and if the method of an assumed ratio had been used, the errors would have affected only the state in which they occurred, but if the Vinton method had been used they might have affected other states.

The only way in which the method of major fractions in the form used in 1840 and revived in 1900 failed to meet the needs of Congress was that it could not furnish for the scrutiny of members a series of tables, each apportioning one more representative than its predecessor. The improvement introduced in 1910 resulted in removing that difficulty. To explain the innovation we must return for a moment to events in the winter of 1900-1901. When the Census Office began the preparation of apportionment tables from the figures of the Twelfth Census, it decided to employ all the methods which had ever been used. For this purpose two sets of tables were constructed, one following the

Vinton method, the other following the method of an assumed ratio in its two forms, rejecting all fractions and rejecting only minor fractions. In preparing the second set of tables the divisors were successive multiples of 500. But a change of 500 in the divisor did not regularly change the number of representatives by one. Often no change, often a change of two, occasionally one of three appeared, and in only one third of the cases did the results of successive tables differ by one representative. Of the two numbers of representatives most considered by Congress, 357 and 386, the former being the existing number and the latter the smallest number under which no state would lose a representative, neither occurred in the second series of tables, following the method of major fractions. Nevertheless, in its decision Congress showed indirectly but convincingly its preference for that method, by selecting the Vinton table for 384 members and adding two representatives for major fractions which, in the table, received no such consideration. Evidently Congress needed a series of tables meeting two requirements, each table apportioning one more representative than its predecessor and every major fraction entitling the state where it occurred to an additional representative; had sought to meet these conditions by translating the results of the Vinton method into those of the method of major fractions; and to do so had broken with the principle on which the Vinton method rests.

To understand how the needs of Congress were met in 1910 we may start with a divisor of six million, which is contained in New York State's population of nine million 1.5 times. Under the method of major fractions a divisor of six million would assign New York two members and every other state one. Let the divisor be reduced unit by unit and each of the series of quotients will slowly increase; when the divisor falls to about five million, the quotient for Pennsylvania rises above 1.5 and that state becomes entitled to a second representative. Let the divisor continue to fall; at about three and three fourths million Illinois becomes entitled to a second representative, and at three and two thirds million New York becomes entitled to a third. By continuing this process a House of any desired size may be apportioned under the method of major fractions. To determine at what ratio the claim of any state to any specified number of members matures and to arrange the ratios for the several states, boundary ratios, as I have ventured to call them, in one series following the order of

decreasing size is a simple problem of elementary mathematics.[4] Then, by assuming as a ratio for division any number between two consecutive boundary ratios, an apportionment table may be constructed which will distribute one more representative than a table constructed by the use of any ratio lying between the next larger pair of boundary ratios. The tables which result lack no characteristic mentioned as desirable and have no characteristic mentioned as undesirable in the congressional debates of the last eighty years. They were accepted in 1911 without any opposition in committee, House, or Senate.

Here the case might be rested and a verdict awaited. But before the bar of theory, or in the judgment of an Association like ours, the verdict of Congress will not be conclusive. I turn, therefore, to an examination of the theoretical advantages and disadvantages of the two competing methods.

The Vinton method starts with an assumed number of representatives and divides the constitutional population of the United States by that number to determine the ratio. This step involves a fundamental theoretical error. It overlooks the crucial fact that seats in the House of Representatives are of two classes, the 48, one for each state, which are guaranteed by the Constitution and are as completely beyond the control of Congress as the seats of the Senators are, and the remainder, the number and distribution of which are under congressional control. The two classes might be named the apportionable and the unapportionable seats. The fact that they are not individually distinguishable has apparently been responsible for the failure to recognize their existence.

To get this theoretical requirement clearly in mind it may be helpful to think of the seats in the House of Representatives as numbered. The first 48 seats, one for each state, would be numbered one to indicate that there is no basis for distinguishing between them. The next seat, numbered 49, would be apportioned to New York, number 50 to Pennsylvania, and so on.

If the first representative falling to Pennsylvania or New York is as irrelevant or disturbing a factor for determining the ratio as the single representative falling to Nevada or Wyoming, then clearly the ratio cannot be found by dividing the constitutional population of the United States by the number of representatives. Nor have I been able to discover any other way by which the pop-

[4] See the writer's letter of explanation in 61st Cong., 3d sess., H. R. 1911, pp. 9-44.

ulation to one representative can be computed from the population of each state and the size of the House.

The effect of this false start is felt in certain anomalies or paradoxes to which the Vinton method gives rise.

A method theoretically sound should apply equally well to the entire range of possibilities whether of interest to Congress or not. The smallest possible House would consist of 49 members, 48 unapportionable and one apportionable, the last obviously going to New York. But if the population of the country is divided by 49 and the population of each state divided by the quotient, as the Vinton method prescribes, New York and Pennsylvania would receive four members apiece, even if no fractions were recognized, and only 41 members would remain to be apportioned among the other 46 states. Near the lower limit of possibilities the Vinton method yields results which are obviously absurd.

A method theoretically sound should never produce the Alabama paradox; under the Vinton method this paradox frequently occurs. In 1901 the majority of the House Committee on the Census recommended that there should be no increase in the size of the House, its number at the time being 357. But in the set of tables we prepared under the Vinton method Colorado received two representatives with a House of 357 or 358 members and three not merely for every higher number but also for every lower number down to 350, where the series of tables began. It was more than difficult, it was impossible, to persuade the House that these results were equitable, and its dissatisfaction with them had no little influence upon its decision to reject the report of the majority and accept that of the minority, substituting another number and table from which the Alabama paradox was absent.

A method theoretically sound must be reversible. Thus, if the House is to contain 435 members, that result might be approached either by adding representatives *seriatim* from the minimum of 48 or by withdrawing them from some number much above 435. A method proposed by Seaton and endorsed by Walker in 1882 was tested by Congress far enough to show that it uniformly favored the populous states and was incontinently rejected on that ground. But if the same method had been reversed and the result approached by successive subtractions rather than successive additions, the outcome would have been just as definitely favorable to the small states.

A method theoretically sound should hold the balance true be-

tween the large and the small states. That the Vinton method has a tendency, slight but persistent, to over-represent the large states is demonstrable. The tendency may be illustrated by the difference between the results of the two methods applied to 357 members. The Vinton method of 1900 gave Colorado and Florida each 2 representatives, while the method of major fractions gave them 3; the Vinton method gave Michigan 12 and Texas 15, while the method of major fractions gave Michigan 11 and Texas 14. Which is correct? The standard is the assumed or computed ratio, the two differing slightly but each indicating 48 representatives to 10,000,000 people. If each of the small states received 3 seats, their ratio would be 56 to 10,000,000, an excess of 8; if each received 2, their ratio would be 38 to 10,000,000, a shortage of 10. Obviously the result giving each of them 3, reached by the method of major fractions, is "as near as may be." Obviously, too, the Vinton method results in under-representing the small states.

For the populous states the outcome depends upon whether the assumed or the computed ratio is made the standard. If the assumed ratio is accepted, then to apportion the smaller numbers to the populous states gives to those states ratios nearer the standard, but if the computed ratio is accepted, there is a very slight balance in favor of the larger numbers. Even if the computed ratio is used, however, the net balance tips in favor of assigning the two seats in question to the smaller states. The result is confirmed by examining the figures for each state separately. Our analysis of this example, then, shows that the Vinton method unjustly, and therefore unconstitutionally, under-represents the small states and over-represents the large ones. These results might be confirmed, if there were time, by making a similar examination of the apportionment of 386 seats in 1900 under the two methods. It would show also that the apportionment of 386 seats by the method of major fractions gives exactly the results which Congress secured illegitimately, as we then believed, by starting with the Vinton table for 384 and adding two for disregarded major fractions. This is an indication that the instinctive judgment of Congress guided them to the right goal although they started on a wrong road.

A method theoretically sound should establish the balancing point between two consecutive numbers of seats, or the size of the fraction entitling the state to another seat, at the middle point or arithmetic mean and not at the geometric mean. This

is a necessary corollary of the preceding position, but needs especial mention, because the use of the geometric mean has recently been advocated. To use it, however, not merely would run counter to the unvarying conviction of Congress that every major fraction gives a valid claim to another seat, it would also result in defeating the main object of the Constitution, which is to hold the scales even between the small and the large states. For the use of the geometric mean inevitably favors the small state. If it were necessary to favor either group, the large states might be entitled to more consideration in the House, because the small states are favored in the Senate. But, fortunately, there is no need to favor either.

As a question of pure mathematical theory, apart from all consideration of motive and from all practical arguments about the judgment of Congress, I find no unanimity of expert opinion in favor of the geometric mean. I have laid the problem before two meetings of the Mathematical Club at Cornell University, and, although no vote was taken, I inferred from the discussion that there was a preponderance of opinion in favor of the arithmetic mean. Thus the theoretical arguments of statists and mathematicians point to the same conclusion to which Congress had already been brought by other considerations and establish my thesis, that the method of major fractions is the correct and constitutional method of apportionment.

The House of Representatives is now more than six times its size before the first census and four times its size immediately after that. Within the half century since the only law passed with a design to check its growth was last put in force, it has increased by more than four fifths. While it has been thus expanding, no similar change, I believe, has occurred in any other representative assembly in this or any other country, with perhaps the exception of the lower House in Austria, where the conditions are unique.

If its present rate of growth should continue for another century, the House would include about 1,400 members. Such an expansion is unlikely and perhaps in the interest of efficiency the increase ought soon to slacken or to stop. From this point of view the change made in 1911 gains new importance. It is now possible for Congress to prescribe, in advance of an approaching census, how many members the House shall contain, to ask the Secretary of Commerce to prepare a table apportioning just that

number in accordance with the method of major fractions, and to report the result to Congress or to announce it by executive proclamation. The experiment which was tried in 1850 and 1860 and which then failed, partly because of inherent defects of method which have since been overcome, and partly because of problems, no longer important, which arose out of the rapid admission of new states and the absence of many members during the Civil War, can now be repeated with more chance of success. As the House is larger by four fifths than it was in 1856 and rapidly growing, the arguments against further increase are stronger.

With a firm grasp of the elements of the problem and a century of experience revealing what Congress regards as the essentials of a sound method, the chance of meeting the conditions are better. Ere long the pressure of opinion within or without the halls of Congress is likely to result in a renewal of the effort to fix the size of the House unalterably. If that effort is made, the removal of the technical difficulties to which your attention has been called this evening will have smoothed the path toward success and have made it more likely that when the change is again introduced it will be permanent.